# Gaussian Beam Propagation

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**Abstract**

This paper describes the propagation of gaussian beams in the absense of turbulence. It is menat a a reference document.

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# 1 Introduction

In this appendix we review the approach of Andrews [1] for describing the Gaussian beam propagation using non-dimensional parameters. First we introduce the approach for ideal Gaussan Beams and then extend it for non-ideal beams that have $M^2$ >1.

# 2 Non-dimentional Beam Parameters

.The radius, or waist, of a Gaussian beam is given by:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

Where $z$ is the distance from the beam waist, and

$$z_R = \frac{\pi w_0^2}{\lambda}$$

is the Rayleigh range. While these parameters describe the Gaussian beam propagation, they are most useful for collimated beams. Here the beam is parameterized as follows:

$$\begin{aligned} F_0 &: \text{The radius of curvature of the wavefront at the aperture} \\ L &: \text{The distance of propagation} \\ W_0 &: \text{The beam radius at the aperture} \\ k = \frac{2\pi}{\lambda} &: \text{The wave number} \end{aligned}$$

At the aperture we have the beam parameters:

$$\begin{aligned} \Theta_0 &= 1 - \frac{L}{F_0} \\ \Lambda_0 &= \frac{2L}{kW_0^2} = \frac{2L}{\left(\frac{2\pi}{\lambda}\right) W_0^2} = \frac{\lambda L}{\pi W_0^2} \end{aligned} \tag{1}$$

We can interpret $\Theta_0 and \Lambda_0$ as describbing the refraction and defraction respectively. For a converging beam:$\Theta_0 < 1$, for a diverging beam $\Theta_0 > 1$, and for a collimates beam $\Theta_0 = 1$. For an (infinite) plane wave, $W_0 = \infty$ and thus, $\Lambda_0$ =0.

At a distance L from the aperture we have:

$$\begin{aligned} \Theta &= \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} = 1 - \frac{L}{F} \\ \Lambda &= \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2} = \frac{\lambda L}{\pi W^2} \end{aligned} \tag{2}$$

and the spot radius at L is determined by:

[1]"Laser Beam Propagation Through Random Media, second edition", Larry Andrews and Ron Phillips, SPIE Press 2005

 

$$\frac{W^2}{W_0^2} = \frac{\Lambda_0}{\Lambda} = \Theta_0^2 + \Lambda_0^2 \tag{3}$$

or

$$W = W_0\sqrt{\Theta_0^2 + \Lambda_0^2} \tag{4}$$

The radius of curveature is given by:

$$F = F_0\frac{(\Theta^2 + \Lambda^2 - \Theta)}{(\Theta - 1)(\Theta^2 + \Lambda^2)} \tag{5}$$

The Irradiance profile is given by:

$$\begin{aligned} I(r,z) &= (\Theta^2 + \Lambda^2)\exp(-\frac{2r}{W}) \\ &= \frac{1}{\Theta_0^2 + \Lambda_0^2}\exp(-\frac{2r}{W_0\sqrt{\Theta_0^2 + \Lambda_0^2}}) \end{aligned} \tag{6}$$

# 3 Non-ideal Beams ($M^2 > 1$)

The previous equations are for diffraction limited beams. Non-ideal beams can be described by the $M^2$ factor. Where ideal beams have $M^2 = 1$and non-ideal beams have $M^2 > 1$. Thus, equations 2 become:

$$\Lambda_0 \rightarrow \Lambda_0 \cdot M^2$$

$$\begin{aligned} \Theta_M &= \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2 \cdot M^4} \\ \Lambda_M &= \frac{\Lambda_0 \cdot M^2}{\Theta_0^2 + \Lambda_0^2 \cdot M^4} \end{aligned} \tag{7}$$

For a focused beam, $\Theta_0 = 0$, we find that:

$$\begin{aligned} \Theta_{L=f} &\rightarrow 0 \\ \Lambda_{L=f} &\rightarrow \frac{1}{\Lambda_0 \cdot M^2} \end{aligned}$$

and

$$\frac{W_M^2}{W_0^2} = 0 + M^4 \cdot \Lambda_0^2 \tag{8}$$

solving for $W_L$:

$$\begin{aligned} W_M &= W_0.M^2 \cdot \Lambda_0 \\ &= W_0.M^2 \cdot \left(\frac{\lambda L}{\pi W_0^2}\right) \end{aligned}$$

Remember that the diffraction limited half-angle for a perfect Gaussian beam is:

$$\theta = \frac{\lambda}{\pi W_0}$$

Thus the spot size for a laser with $M^2 > 1$ is:

$$W_M = M^2\,(\theta \cdot L) = M^2 \cdot (\text{diffraction limited spot size})$$

For a collimated beam, at long range , $\Theta_0 = 1, \Lambda_0 \gg 1$, we find that

$$\frac{W_M^2}{W_0^2} \quad \approx' \quad M^4 \cdot \Lambda_0^2 = M^2\,(\theta \cdot L) \tag{9}$$

Which is the same as we found in the focused case. Alternatively, we could start with Equation 1:

$$\begin{aligned} \Theta_L &\rightarrow \frac{1}{\Lambda_0^2 \cdot M^4} \\ \Lambda_L &\rightarrow \frac{1}{\Lambda_0 \cdot M^4} = \frac{\lambda L}{\pi W_{L,M^2}^2} \end{aligned}$$

Solving for $W_{L,M^2}$ , we find:

$$\begin{aligned} W_M &= \sqrt{\frac{2L}{k\left(\frac{1}{\Lambda_0 \cdot M^4}\right)}} \\ &= \sqrt{\frac{2\Lambda_0 M^4}{k}} \\ &= M^2\sqrt{\frac{2\lambda L}{\pi k W_0^2}} \\ &= M^2 \cdot W_L \end{aligned}$$

Which is the expected result. The radius of curveature is given by:

$$F_M = F_0 \frac{(\Theta_M^2 + \Lambda_M^2 - \Theta_M)}{(\Theta_M - 1)(\Theta_M^2 + \Lambda_M^2)} \tag{10}$$

Where, $\Theta_M$ and $\Lambda_M$ are given by Equation 7. Similarly, the Irradiance profile is given by:

$$I(r,z) \quad = \quad (\Theta_M^2 + \Lambda_M^2)\exp(-\frac{2r}{W_M}) \tag{11}$$

 